Optimal Control of a Somersaulting Platform Diver: A Numerical Approach

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Abstract

In this paper we study the somersaulting maneuver of a platform diver and give an effective numerical approach for obtaining an optimal solution for it. Modelling the diver as a planar system of interconnected multibodies, we prove controllability in a sense dictated by the problem. We set up a time optimal control problem with state and control constraints and solve it using our numerical approach. The numerical solution agrees well with motions executed by professional divers.

1. Introduction

In this paper we study various issues associated with the somersaulting maneuver, without twist, of a platform diver, with the aim of deriving a robust and efficient numerical approach to determine optimal diving motions. The numerical approach that we suggest is quite general and can be easily extended to other planar interconnected systems with angular momentum conservation constraints. Such systems are useful in a number of applications, for example, motion planning of manipulators mounted on space vechicles, astronauts' reorientation maneuvers in space, deployment maneuvers for multibody antennas connected to spacecraft, monoped in flight etc. See [3–6] and the other references given there for details on these and other applications.

The dynamic equations that model a somersaulting diver are derived in section 2. A number of special issues make platform diving quite different from reorientation maneuvers of other applications. These issues are discussed in section 3. We have taken useful guidelines from the excellent qualitative study of Frohlich on platform diving [2]. These issues lead to various constraints on the dive trajectories.

In section 4 we prove controllability in a sense which is appropriate for the diver's problem. Our ideas on this issue are very different and less sophisticated than the controllability results proved by Reyhanoglu and McClamroch [5] for systems with zero angular momentum.

Since a diver wishes to maximize the number of somersaults that can be made in a given time, it is appropriate to formulate and slove a time optimal control problem. In section 5 we suggest two numerical approaches for solving the problem. The first approach uses a discretization of the control vector consisting of the internal torques. This is similar to the approach taken by Fernandez, Gurvits and Li [1] to solve the falling cat problem. We faced a lot of difficulties in solving the diver's problem using this approach. The second approach is based on discretizing the state trajectory. This approach had an excellent control over the motion, and led to the quick obtainment of optimal motions.

In section 6 we consider two diving maneuvers to illustrate our approach: a forward somersault and a backward somersault. The second numerical approach gave nice solutions that corresponded well with motions actually performed by professional divers. Concluding remarks and possible future extensions of our work are indicated in section 7.

2. Modelling

A diver doing somersaults, without twist, is modelled as a planar system made up of n rigid rectangular links connected together by revolute joints to form an open kinematic chain. The choice of n depends on the particular maneuver executed and the preciseness with which the bending of the body is to be modelled. For a backward somersault a two link model will suffice. Divers doing this somersault seem to bend only the hip joint. For a forward somersault at least a four link model is needed to include bending at the knee, hip and shoulder joints. A more accurate model can include the elbow joint and two more joints between the hip and the shoulder to account for the flexibility of the body.

Consider the schematic in fig.1. Let O represent the center of mass of the system, O_i denote the center of mass of the i-th link, and A_j denote the j-th pin joint. It is not difficult to see that gravity does not contribute in any way to the somersaulting operation. It only decides the total time period available to do the somersault. (A diver diving from a height of 10 meters has about 1.5 seconds to work with.) Hence for our purpose O can be assumed to be fixed.

We now derive the dynamic equations associated with the above model. The derivation uses standard ideas. Yet we do give some details which are important from the viewpoint of computational efficiency. The numerical solution that we give later requires numerous evaluations of the dynamic equations and, the difference between a casually written set of equations and a carefully written set can be as large as a factor of ten in cpu time.

Let : m_i = mass of the i-th link; l_1 = distance from O_1 to A_1 ; for $i \geq 2$, l_i = length of the i-th link; $\tilde{l}_1 = 0$; for $i \geq 2$, $\tilde{l}_i$ = distance from A_{i-1} to O_i ; and I_i be the inertia of the i-th link. Also let: $p \in R^n$ be the vector of (absolute) angles made by the links with the positive horizontal axis; and $q \in R^n$ consist of

q_1, the angle made by the first link with the positive horizontal axis and $q_2, \cdots, q_n$, the relative angles at the $(n-1)$ joints. Let T be the $n \times n$ lower triangular matrix with all elements on and below the diagonal equal to one. Then p and q are related by

$$p = Tq \ . \tag{2.1}$$

Both p and q are independent choices for the generalized coordinates. The choice of p is good from the view point of computational efficiency, whereas choosing q exposes the specialities of the dynamics in a clear way.

The kinetic energy of the system can be written as

$$K = \frac{1}{2}\dot{p}' D(p)\dot{p} \tag{2.2}$$

where: $\dot{p} = dp/dt$; prime denotes transpose;

$$D_{ij}(p) = e_{ij}\cos(p_i - p_j) + I_i\delta_{ij} \ ; \tag{2.3}$$

$\delta_{ij} = 1$ if $i = j$, 0 otherwise; $e_{ij} = \sum_{k=1}^{n} m_k c_{ki} c_{kj}$; $c_{kr} = (l_r - b_r)$ if $1 \leq r \leq k-1$, $(\tilde{l}_k - b_k)$ if $r = k$, $-b_r$ if $k+1 \leq r \leq n$; $b_r = (m_r/M)\tilde{l}_r + l_r a_r$; $a_r = (m_{r+1} + \cdots + m_n)$ if $r < n$; and $a_n = 0$. Let $u \in R^{n-1}$ denote the vector of torques applied at the $(n-1)$ joints. Using (2.2) and (2.3) the Euler-Lagrange equations can be written as

$$D(p)\ddot{p} + f(p, \dot{p}) = Bu \tag{2.4}$$

where: B is an $n \times (n-1)$ matrix defined by $B_{ij} = -1$ if $i = j$, 1 if $i = j+1$, 0 otherwise;

$$f_k(p, \dot{p}) = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho(i,j,k)\dot{p}_i\dot{p}_j \ ;$$

$\rho(i,j,k) = \lambda(j,k,i) + \lambda(i,k,j) - \lambda(i,j,k)$; and, the λ function is defined by $\lambda(i,j,k) = -e_{ij}\sin(p_i - p_j)$ if $k = i$, $e_{ij}\sin(p_i - p_j)$ if $k = j$, 0 otherwise.

Let us now analyze the dynamics using q as the vector of generalized coordinates. Classify q as $q = (\theta, \psi)$ where $\theta \in R$ and $\psi \in R^{n-1}$. Using (2.1)–(2.2) the kinetic energy can be written as $K = 0.5\dot{q}'\tilde{D}(q)\dot{q}$ where $\tilde{D}(q) = T'D(Tq)T$. Since, by (2.3), $\tilde{D}$ is independent of θ, hereafter we will refer to $\tilde{D}(q)$ as $\tilde{D}(\psi)$. Let

$$\tilde{D}(\psi) = \begin{pmatrix} \alpha(\psi) & \beta'(\psi) \\ \beta(\psi) & \Gamma(\psi) \end{pmatrix}$$

where $\alpha(\psi) \in R$, $\beta(\psi) \in R^{n-1}$ and $\Gamma(\psi) \in R^{(n-1)\times(n-1)}$. When we write the Euler-Lagrange equations for q, the equation corresponding to $q_1 = \theta$ leads to

$$\alpha(\psi)\dot{\theta} + \beta'(\psi)\dot{\psi} = c \tag{2.5}$$

where c is constant during motion. Equation (2.5) expresses conservation of angular momentum (c). Using (2.4) and (2.5) the equations for ψ can be written as

$$J(\psi, c)\ddot{\psi} + F(\psi, \dot{\psi}, c) = u \ . \tag{2.6}$$

We will not say much about J and F since they are never used by the computational algorithm to be given later. We will only point out the following useful property: $F(\psi, \delta\dot{\psi}, \delta c) = \delta^2 F(\psi, \dot{\psi}, c)$.

3. Issues in Platform Diving

Frohlich [2] gives an excellent qualitative study of various issues associated with platform diving. We have taken several useful hints from this study. These issues make platform diving quite different from other types of reorientation maneuvers. Let us now go into these issues in detail.

While just leaving the platform, the diver has the ability to generate a good starting velocity, v^0 that aids the somersault. Frohlich points out various ways that divers employ to generate these velocities, e.g., by the throwing of the arms. In our formulation, therefore, we take v^0 as a variable vector subject to limits on its magnitudes:

$$\dot{q}(0) = v^0 \ ; \quad |v_i^0| \leq V_i \ \ \forall i \ . \tag{3.1}$$

While just leaving the platform the diver is usually in a slightly curled up position that deviates from the vertical stretched up position, $\theta = \pi/2$, $\psi = 0$. This helps the diver to reach quickly the fully coiled configuration during the initial phase of the somersault. we express this factor as

$$q(0) = q^0 \ . \tag{3.2}$$

The choice of q^0 depends on the type of dive performed. Typical values of q^0 will be mentioned in section 6.

The aim of the diver is to perform the prescribed number of somersaults and come to an inverted stretched position at the end of the dive when he just enters the water. The final configuration constraint can be expressed as

$$q(t_f) = q^f \ , \tag{3.3}$$

where t_f denotes the final time and q^f depends on the type of dive and the number of somersaults to be made. For example, for k forward somersaults we require $q^f = (2k\pi + \pi + \pi/2, 0)$.

Because initial velocities are present the angular momentum, c during the motion is typically non-zero. (There is a reason for requiring c to be well away from zero; see the discussion of 'phase 2' given below.) Thus putting a hard constraint on $\dot{q}(t_f)$ such as $\dot{q}(t_f) = 0$, in our formulation is not a good idea. It should be mentioned that only the best divers can control the final residual velocities well; the velocities get damped out soon after entering the water.

During the motion u and ψ have to obey bound constraints. The constraints on u come from limits on the torque magnitudes that can be generated. The constraints on ψ arise because of the physical organization of the limbs. For example, there can be only backward bending at the knee joint and, even that bending is allowed only up to a value of π radians (at which point, the leg touches the thigh). The above constraints on u and ψ are point-wise in time:

$$|u_i(t)| \leq U_i \;\; \forall\, i, t \qquad (3.4)$$

$$\Psi_i^{\min} \leq \psi(t) \leq \Psi_i^{\max} \;\; \forall\, i, t \qquad (3.5)$$

Let $\mathcal{U}$ and $\mathcal{X}$ be, respectively, the rectangloids (with non empty interior) in the u and ψ space defined by the above inequalities. Note that $\mathcal{U}$ also contains the origin in the interior.

Studying actual somersaults executed by professional divers reveals three distinct phases of a somersault. The first phase results in the diver achieving a coiled up configuration. If $[0, t_1]$ denotes the period for phase 1 then the aim of phase 1 is to achieve

$$\theta(t_1) = \theta^1, \;\; \dot{\theta}(t_1) = \dot{\theta}^1, \;\; \psi(t_1) = \psi^1, \;\; \dot{\psi}(t_1) = 0 \,. \qquad (3.6)$$

Of course, if c is decided by the initial velocity v^0 and (2.5) then $\dot{\theta}^1$ is fixed.

In the second phase the diver remains in the coiled up configuration and somersaults. Hence, in this phase only θ is varying. If $[t_1, t_2]$ denotes the phase 2 period then

$$\psi(t) = \psi^1, \;\; \dot{\psi}(t) = 0 \;\; \forall\, t \in [t_1, t_2] \,. \qquad (3.7)$$

The following result gives an analytical solution of phase 2. Its proof follows directly from (2.5), (2.6) and (3.6).

Proposition 1 *The phase 2 constraint,* (3.7) *can be acheived if and only if*

$$u(t) = F(\psi^1, 0, c) \;\; \forall\, t \in [t_1, t_2] \,, \qquad (3.8)$$

a constant function of time where F is as in (2.6). *Further, in phase 2 we have*

$$\theta(t) = \theta^1 + \dot{\theta}^1(t - t_1) \;\; \forall\, t \in [t_1, t_2] \,. \qquad (3.9)$$

The actual determination of the phase 2 torque, $F(\psi^1, 0, c)$ can be done easily without resorting to F. We can simply set: $q = (\theta^1, \psi^1)$, $\dot{q} = (\dot{\theta}^1, 0)$, $p = Tq$, $\dot{p} = \dot{\theta}^1(1, \cdots, 1)'$, $\ddot{p} = 0$ and solve the overdetermined (but consistent) system of linear equations, (2.4) for u. (Because of the nice structure of $\tilde{B}$, this linear system can be easily solved in closed form.)

Another remark concerning phase 2 is worth mentioning here. Suppose c is a fixed level of angular momentum. If somersaulting is to be fast then $\dot{\theta}^1$ should be as large as possible. Since, by (2.5) and (3.6), $\dot{\theta}^1 = c/\alpha(\psi^1)$, the vector ψ^1 should be chosen in such a way that $\alpha(\psi^1)$ is as small as possible. The coiled up position of the diver corresponds to this optimal choice in which all the links come together as close as possible.

In the third phase the diver starts from the configuration, $\theta = \theta^1 + \dot{\theta}^1(t_2 - t_1)$, $\psi = \psi^1$ and uncurls to reach the final configuration set by (3.3). In the examples to be presented later we will take full advantage of the above three phase ideas to generate good somersault trajectories.

4. Controllability

Recently Reyhanoglu and McClamroch [5] have given several useful results on the controllability of planar multibody systems with angular momentum preserving controls. They consider systems of the type considered in this paper, but take the angular momentum, c to be zero. Reyhanoglu and McClamroch show (by actually constructing $u(\cdot)$) that if the number of links is three or more then any $(\theta^0, \psi^0, \dot{\psi}^0)$ can be transferred to any equilibrium, $(\theta^f, \psi^f, 0)$ in arbitrarily small time. (Using the property, $F(\psi, -\dot{\psi}, 0) = F(\psi, \dot{\psi}, 0)$, this result can be easily extended to include transfer from any $(\theta^0, \psi^0, \dot{\psi}^0)$ to any $(\theta^f, \psi^f, \dot{\psi}^f)$.)

The problem considered in this paper is such that 'full controllability' is not necessary. Because there is a lot of freedom in choosing v^0, and $v^f = \dot{q}(t_f)$ is free, finding $u(\cdot)$ that transfers q from any q^0 to any q^f is easy. In fact, as the following result and its proof show, the partial controllability requirement also makes the achievement of the control and state constraints in (3.4) and (3.5) easy.

Proposition 2 *There exists $u(\cdot)$ which transfers the system* (2.5)–(2.6) *from any q^0 to any q^f (of course, q^0 and q^f should satisfy the bounds indicated in* (3.5)) *in finite time, while satisfying* (3.1) *and* (3.4)–(3.5).

Proof Let $q^0 = (\theta^0, \psi^0)$ and $q^f = (\theta^f, \psi^f)$. Choose any $t_f > 0$, and then choose any twice differentiable function, $\psi : [0, t_f] \rightarrow \mathcal{X}$ such that $\psi(0) = \psi^0$ and $\psi(t_f) = \psi^f$. From (2.5) we have

$$\theta(t_f) = \theta^0 + c\int_0^{t_f} (1/\alpha) dt \; - \; \int_0^{t_f} (\beta' \dot{\psi}/\alpha) dt \;\; . \qquad (4.1)$$

Note here that the diver model is such that $\alpha(\psi)$ is always sufficiently positive (this follows from the positive definiteness of $D(p)$ and $\widetilde{D}(\psi)$) and so division by it is well defined. By choosing the right value for c (this can be done by choosing $\dot{\theta}(0)$ correctly) we can achieve $\theta(t_f) = \theta^f$. This also determines $\theta(\cdot)$. Having chosen $q(\cdot)$ this way, determine $u(\cdot)$ from (2.6). Finally, if the $(u(\cdot), q(\cdot))$ violates (3.1) and/or (3.4), then expand the time scale appropriately so as to satisfy (3.1) and (3.4). Note here that this operation will not disturb (3.5) and the end constraints defined by q^0 and q^f.

5. Numerical Optimal Control

A diver is interested in completing the somersault in a limited time. Thus, setting up and solving a time optimal control problem is appropriate. The problem is:

$$\begin{array}{lll} \text{minimize} & t_f & \\ \text{subject to:} & (3.1) - (3.5) & \text{and,} \\ & (2.5) - (2.6) & (\text{or, } (2.1), (2.4)) \end{array} \qquad (5.1)$$

Because of the complexity of the nonlinearities and constraints involved, the problem is non-trivial to solve, even numerically. Our aim is to give an efficient numerical approach to find an approximate solution of (5.1).

Using the special nature of the dynamics and constraints, problem (5.1) can be equivalently written as a fixed time problem as follows. Fix any positive value for t_f, say 1 and solve:

$$\begin{array}{ll} \text{minimize} & f = \max\{f_1, f_2\} \\ \text{subject to:} & (3.2)-(3.3),(3.5) \quad \text{and,} \\ & (2.5)-(2.6) \quad (\text{or, } (2.1),(2.4)) \end{array} \tag{5.2}$$

where

$$f_1 = \max_i \left(\frac{v_i^0}{V_i}\right)^2 \quad \text{and} \quad f_2 = \max_i \left(\operatorname*{ess.sup}_{t\in[0,t_f]} \frac{|u_i(t)|}{U_i}\right) \tag{5.3}$$

Thus the minimum time problem is equivalent to minimizing, in a particular sense, the magnitudes of torques and velocities in a fixed time framework.

To numerically solve any one of the above problems we have to make an approximation to convert it into a finite dimensional optimization problem. We tried two approaches. In the first approach we discretized the control trajectory, $u(\cdot)$ and used problem formulation (5.1). Various ways of discretization are: (i) divide $[0, t_f]$ into a fixed number of equal intervals and take u to be a different polynomial function in each interval; (ii) approximate u as a truncated Fourier series; and, (iii) divide $[0, t_f]$ into a fixed number of unequal intervals and assume u to be constant in each of these intervals. Because all linear systems and a class of nonlinear systems exhibit a bang-bang time optimal control trajectory, we preferred to use the third way of discretization. Here the optimization variables are: the initial velocity vector v^0, the control switch times, and the piecewise constant control magnitudes.

The numerical solution was obtained as follows. First, simple bounds on variables (e.g., those defined by (3.1) and (3.4)) were treated by introducing unconstrained variables using a transformation. For example: $x \geq a$ was taken care of by setting $x = a + y^2$ where y is unconstrained; $a \leq x \leq b$ was treated by setting

$$x = (b+a)/2 + ((b-a)/2)y/\sqrt{1+y^2} \tag{5.4}$$

where y is unconstrained, etc. The equality constraints, (3.2), (2.1) and (2.4) were eliminated by numerically integrating (2.4) from the starting point defined by (2.1) and (3.2) every time the objective function was to be evaluated. A penalty function approach was used to deal with the remaining constraints, i.e., (3.3) and (3.5). The resulting unconstrained nonlinear optimization problems were solved using an efficient implementation of conjugate-gradient and quasi-Newton methods.

In spite of spending a lot of time and effort, we could not achieve much using the above approach. The final state constraint, (3.3) was difficult to achieve. Because there was little direct control over the state trajectory, satisfying (3.5) was also hard. Furthermore, the numerical integration of (2.4) was very sensitive to changes in the control magnitudes and switching times, thus leading to severe errors in the objective function evaluation. The computing times involved were also high, say in the order of hours on an Intel 80486 based machine. Because of these reasons we discarded the above approach.

Recently, Fernandez, Gurvits and Li [1] have given a similar approach to solve the falling cat problem. Their discretization of u uses truncated Fourier series. We have briefly experimented with truncated Fourier series for our problem. The performance remained as bad as before.

In the second approach, which turned out to be very successful, we discretized q and used problem formulation (5.2). We divided $[0, t_f]$ into a finite number of equal intervals and took, as variables, the values of q at the interior time grid points together with v^0 and v^f. Given a set of values of these variables, there is a unique cubic spline fit, $q(\cdot)$ which is twice differentiable and satisfies (3.2)-(3.3). As in the first approach the resulting optimization problem was solved using a penalty function method. To ensure the satisfaction of the dynamic equations we included, in the penalized objective function, an appropriately weighted integral of the square of the error in satisfying (2.5). To evaluate f as defined by (5.2) and (5.3) we required u, which we obtained via (2.1) and (2.4). The state constraints in (3.5) were also integrated into the objective function using penalties. Because (3.5) corresponds to simple bounds, the trick in (5.4) was used on the q values at the time grid points. This gave an effective control over the satisfaction of (3.5) at other times too. The bound constraint in (3.1) was also handled in a similar way. A combination of conjugate gradient method and trust region method for nonlinear least squares, was used with this approach.

Both approaches handle the initial conditions easily. The first approach also directly takes care of the satisfaction of the dynamic equations and the control constraints. However, it has a lot of difficulty in handling the state constraints (both the point-wise and the final ones). On the other hand, the second approach has excellent control over the state constraints. Its challenge lies in the satisfaction of the dynamic equations. In a number of numerical experiments on the second approach, we observed that, even if the initial choice of the variables was such that the dynamic equations were violated badly, just a few optimization iterations lead to an excellent satisfaction of the dynamic equations. Overall, this approach seems to be the best suited for solving the diver's problem.

6. Numerical Results

We have used the ideas outlined in the previous sections to solve two diving maneuvers: a forward somersault and a backward somersault. While at least a four link model is necessary for the former, a two link model is sufficient for the latter. Thus, the choices of links are as follows: leg, thigh, body and arm for the first problem; and, leg + thigh, body + arm for the second problem. For both problems the links are indexed starting from the leg and moving upwards; in particular, $q_1 = \theta$ is always the angle made by the leg with the positive horizontal axis. We assume that only one somersault is made by the diver (i.e., θ roughly changes by 3π radians). Because an analytical solution of phase 2 is available (proposition 2) more somersaults can be easily included.

The second numerical approach of section 5 was successfully used to solve (5.2). We solved (5.2) instead of (5.1) not for any numerical convenience, but simply because good estimates of the U_i were not available. Because we could replace the f in (5.2) by $\delta \cdot f$, where $\delta > 0$, without affecting the solution obtained, all that we needed to know were good guesses for the relative weights between the various V_i's and U_i's. We were able to make these guesses using arguments based on maximum velocities that can be generated by the limbs. Thus we fixed t_f at values achieved by professional divers, solved (5.2) and then finally checked if v^0 and u are physically reasonable. The guesses for the U_i and the V_i, together with some other data associated with the two dive problems are given in the following table.

		Forward Somersault	Backward Somersault
q^0	deg.	(90,-20,45,20)	(90,0)
q^f	deg.	(630,0,0,0)	(-450,0)
V	rad/sec	(15,15,15,15)	(5,5)
U	N–m	(1000,5000,1000)	(3000)
Ψ^{min}	deg	(-180,-20,0)	(-20)
Ψ^{max}	deg	(0,180,180)	(180)
t_f	sec	0.94	1.15

We have used the three phase idea outlined in section 3 to simplify the numerical solution. The solution was obtained in two steps. In the first step we guessed reasonable values for θ^1, $\dot{\theta}^1$, ψ^1, t_1 and t_2 (see (3.6)-(3.7)), and solved two independent optimal control problems corresponding to phase 1 and phase 3. In the second step we solved the full diver's problem by keeping the three phase structure and letting θ^1, $\dot{\theta}^1$, ψ^1, t_1 and t_2 free, and using the solutions obtained in the first step to start the numerical solution. The analytical solution of phase 2, given by proposition 1 was used. We believe that the solution obtained this way (i.e., subject to the three phase structure constraint) is an excellent suboptimal solution of (5.2).

The procedure outlined above was coded in Fortran and run on an Intel 80486 based machine. Numerical solution was continued until the objective function could not be decreased any more. The total cpu time required to solve each of the two dive problems was about ten minutes.

For lack of space we give details of the final solution only for the forward somersault, which is adequately described by figs.2–6. The dive motions correspond reasonably well with those performed by professional divers. This was confirmed by comparing our trajectories with the dives of Mingxia Fu of the Chinese team in the women's 10 m platform dive event of the recently concluded Olympics'92. While the torque and initial velocity magnitudes appear reasonable to us, only a biomechanics expert can comment on their physical realizability. It should be noted that the largest magnitudes of torque occur during phase 1, and that the torques associated with phase 2 are quite small. The torque trajectories have a non-trivial shape and, nicely approximating them by either piecewise constant time functions or truncated Fourier series is not possible unless the number of variables used is large. This may be one of the reasons why the first numerical approach of section 5 did not work well.

7. Conclusion

In this paper we have given an efficient and robust numerical approach for the determination of an optimal somersaulting motion of a platform diver. In the process of doing this we have discussed various important issues associated with platform diving. There are three limitations of our model which are worth considering in future models. They are as follows. (1) The effect of friction at the joints is neglected. (2) The effect of configuration dependent damping has not been included; for example, when the lower leg is brought close to the thigh in the curled up position, there is substantial damping. (3) A study of actual dives executed by professional divers shows that the diver holds his legs tightly using the arms to maintain his configuration during phase 2; it is unclear whether this change over to a closed chain model helps in improving a dive.

Our numerical approach is quite general and can be easily extended to other multibody systems. We are currently using our approach to solve the following problems: reorientation maneuvers in space; falling cat; and, three dimensional diver's motion with twist.

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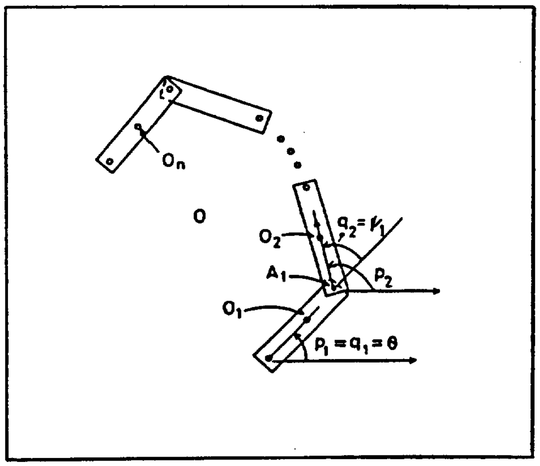

Figure 1. The diver modelled as a planar system

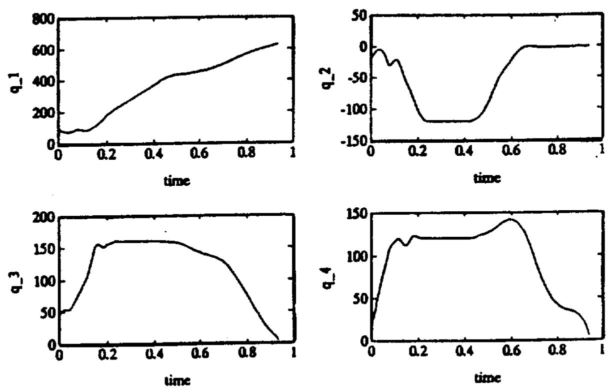

Figure 2. Forward somersault: q trajectories
(angles are in degrees)

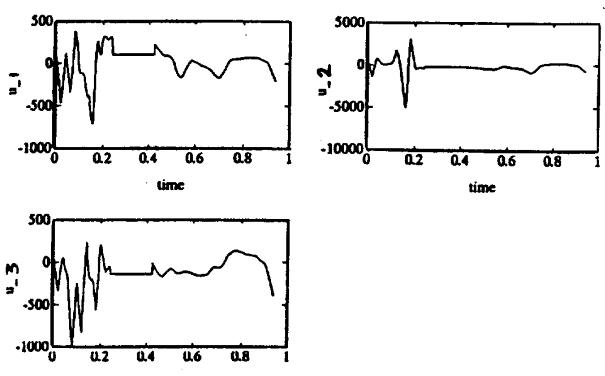

Figure 3 Forward somersault: u trajectories
(torques are in N-m)

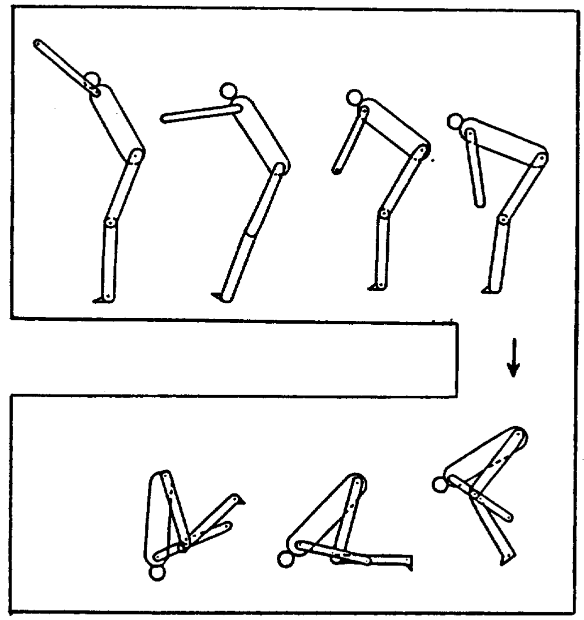

Figure 4. Forward somersault: A sequence of diver configurations during phase 1

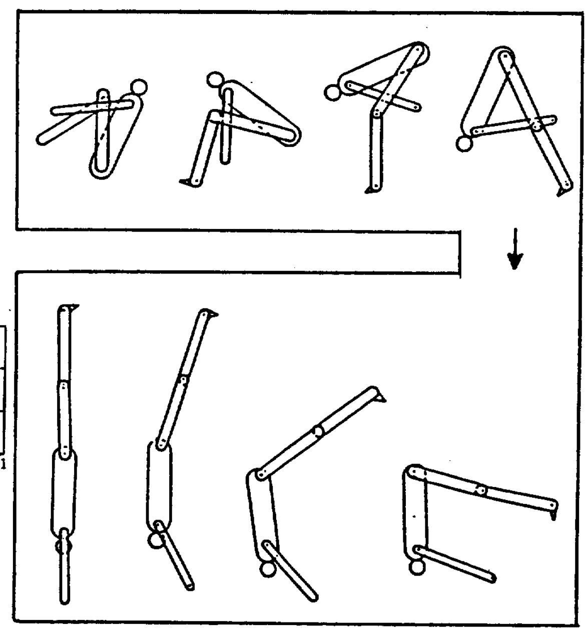

Figure 5. Forward somersault: A sequence of diver configurations during phase 3